Impact resistance and energy absorption of regular and functionally graded hexagonal honeycombs with cell wall material strain hardening

D. Mousanezhad a, R. Ghosh a, A. Ajdari b, A.M.S. Hamouda c, H. Nayeb-Hashemi a, A. Vaziri a,*

a Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA, USA
b Department of Mechanical Engineering, Northwestern University, Evanston, IL, USA
c Mechanical and Industrial Engineering Department, Qatar University, Doha, Qatar

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This paper highlights the effects of cell wall material strain hardening and density functional gradation (FG) on in-plane constant-velocity dynamic crushing response and impact behavior of hexagonal honeycombs. Results show that cell wall material strain hardening influences the distinct deformation modes induced by crushing velocity generally observed in regular hexagonal honeycombs. This is seen by a delay in the onset of localized deformation up until intermediate crushing velocities after which localization becomes dominant smearing out differences brought about by cell wall material strain hardening (plasticity convergence). In addition, during the impact loading on regular honeycombs, it was found that increasing the cell wall material strain hardening decreases the rate of gain of maximum crushing strain with increments in initial kinetic energy of impact. On the other hand, introducing FG brings about new deformation patterns due to changes in material distribution and preferential cell wall collapse of the weaker members. Interestingly, although the dynamic localization effect at higher crushing velocities observed earlier was not found to be particularly affected by FG, gradient convergence (i.e. smearing out the effects of FG due to higher velocities analogous to plasticity convergence) was not observed. On the contrary, gradient convergence emerged at higher impacting velocities primarily brought about by a combination of initial deformation localization and its subsequent advancement into FG region ahead. The kinetic energy threshold for the emergence of this gradient convergence effect was found to be considerably delayed by cell wall material strain hardening.

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1. Introduction

The unique mechanical properties and multifunctional advantages offered by low density cellular structures make them popular candidates for a wide range of applications including thermal insulation, packaging, shock mitigation and myriad aerospace applications [1]. Among numerous ways in which cellular construction can be incorporated into structural systems, metallic sandwich panels with low density cellular core construction are particularly convenient to design, fabricate and deploy across a wide range of operating conditions. The applications of such sandwich panels in blast mitigation and projectile impact have been under extensive research during the last decade, showing that metallic sandwich panels have in general, superior energy absorption capacity and impact resistance compared to solid plates made of the same material with equivalent total mass [2–24].

It is well known that when a cellular structure is loaded quasi-statically (in-plane loading), the stress–strain curve of the structure contains three distinct regimes: linear-elastic, plateau, and densification [1,25–27]. In contrast, under dynamic loads the response of cellular structures is mainly dominated by localized phenomena such as buckling and micro-inertial resistance [9,17,28–31]. In addition, inelastic properties of the material can add significant complexity to the mechanical response of the structure. Papka and Kyriakides [32] were among the pioneers in considering the effects of strain hardening of the cell wall material of cellular structures using a combination of experiments and analysis. Several other investigations [33–36] have focused on understanding the role of cell wall material strain hardening on the behavior of foams at small strains in the quasi-static regime. One of the recent studies in this area by Mangipudi et al. [37] investigated the microstructural origin of strain hardening in two dimensional open-cell metal foams. They found that there exist two sources contributing to strain hardening in cellular materials – strain hardening of the cell wall material (micro-scale), and geometric hardening as a result of strut reorientation (meso-scale). Assuming small deformations, they derived...
a scaling relation between the hardening tangent modulus of the material, $H$, to that of a regular hexagonal honeycomb structure, $H^*$, given by $H^*/H = E^* / E = (4 / \sqrt{3})(t/a)^2$, where $E^*$ and $E$ are Young's moduli of the structure and cell wall material, $t$ is the thickness and $a$ is the length of the cell walls. Note that the hardening tangent modulus is the post-yield slope of the stress-strain curve of a material in the case of linear strain hardening material behavior. We will use this relation in Section 3 of this paper to estimate the impact response of regular hexagonal honeycombs.

In addition to including the effect of cell wall material strain hardening of regular hexagonal honeycombs, we also study the effect of changing the material distribution on dynamic response of honeycombs. This latter, a novel class of cellular structures inspired from nature [38,39], called functionally graded (FG) honeycombs, can be generated through the variations in cell geometry (i.e. cell size, shape, wall thickness, etc.). These variations in material distribution result in a corresponding functional gradation of properties throughout the structure. Interestingly, these resultant variations can be exploited to control the energy absorption capacity of cellular materials [12,40–48]. Recently, Ajdari et al. [28] studied the in-plane dynamic crushing of two dimensional honeycombs with both regular hexagonal and irregular arrangements brought about due to both defects and irregular microstructure as well as FG structures. However, their study assumed an elastic-perfectly plastic (no strain hardening) material behavior of the cell wall material neglecting strain hardening. In this paper, we investigate the effect of cell wall material strain hardening on dynamic crushing and impact mechanics of regular and FG hexagonal honeycomb structures.

The paper is organized as follows: Section 2 describes the details of the finite element (FE) models. Sections 3 and 4 are respectively devoted to compare the behavior of regular and FG hexagonal honeycombs with and without material strain hardening under dynamic crushing and impact loading using detailed FE models. Theoretical estimations for the response of regular hexagonal honeycombs with cell wall material strain hardening under rigid plate impact are also provided in Section 3. Conclusions are drawn in Section 5.

2. Finite element simulations

Three dimensional models of regular hexagonal honeycombs were meshed using 4-node shell elements (S4R) using commercially available FE software ABAQUS 6.11-2 (SIMULIA, Providence, RI) which was also used to carry out all the simulations. Mesh convergence study was also performed in order to remove any mesh dependence on final results. The following elastic-linear strain hardening model was assumed for the cell wall material of the structures:

$$
\sigma = \begin{cases} 
\sigma_Y, & \epsilon \leq \frac{\epsilon_Y}{2} \\
\sigma_Y + H(\epsilon - \frac{\epsilon_Y}{2}), & \epsilon > \frac{\epsilon_Y}{2}
\end{cases}
$$

where $\sigma$ and $\epsilon$ are the stress and strain, $E$ and $H$ are Young's modulus and hardening tangent modulus, $\sigma_Y$ and $\epsilon_Y$ ($=\sigma_Y / E$) are the yield stress and yield strain of the cell wall material. For convenience, the hardening tangent modulus is normalized by Young's modulus to arrive at a non-dimensional cell wall material strain hardening parameter, $\bar{H} = H/E$. The cell wall material was assumed to have Young's modulus, $E = 70$ GPa, yield strength, $\sigma_Y = 130$ MPa, Poisson's ratio, $\nu = 0.3$, and density, $\rho = 2700$ kg/m$^3$, which are close to aluminum till yielding commences. The relative density of a regular hexagonal honeycomb, $\rho_c$, is defined as the ratio of the in-plane area of the cell walls to the total in-plane area of the structure and obtained as $\rho_c = (2/\sqrt{3})(t/a)^2$, where $t$ is the thickness and $a$ is the edge length of the hexagons [1].

Note that although the paper deals with dynamic loadings, no dynamic effects (e.g. strain-rate dependence) were considered for the cell wall material of the honeycombs. This is a reasonable assumption for the current work since it is well known that metallic sandwich plates under shock loadings exhibit negligible material strain-rate dependence for steels [16,18,24]. Furthermore, even for aluminum foils, experimental results show that moderate strain-rate sensitivity becomes significant only at strain-rates of the order of $\sim 10^2 - 10^3$/s [49,50]. Interestingly, some aluminum alloys are insensitive to strain-rate even at high strain-rates [50]. Furthermore, Zhao and Gary [49] showed that only the out-of-plane crushing behavior of aluminum honeycombs is affected by loading rate. In this paper, we have neglected strain-rate sensitivity of the cell wall material on the overall dynamic response of the honeycombs.

In order to simulate the constant-velocity crushing load, two rigid flat plates were first tied (i.e. welded) to the honeycomb structure on the top and the bottom. The top one was then imparted a constant downward velocity, $V$, and the bottom one was fixed. Periodic boundary conditions were assumed for the left and right sides of the structure [51]. To prevent out-of-plane buckling, the out-of-plane degrees of freedom of the model were constrained. A dimensionless normalized velocity, $\bar{V}$, is defined to address the inertial effects, where $\bar{V} = V/(c_0 \sqrt{\rho})$, and $c_0 = (E/\rho)^{0.5}$ is the elastic wave speed in the cell wall material [17,28].

Note that periodic boundary conditions, which were assumed instead of traction free conditions, were imposed on the left and right sides of the structure to eliminate boundary effects, although traction free boundary conditions are indeed reasonable [52]. To simulate in-plane rigid plate impact response of hexagonal honeycombs, we developed a FE model which contains a rigid flat plate with mass $M$ impacting the honeycomb with an initial velocity, $V_0$. During the impact, as the rigid plate crushes the structure, its initial kinetic energy, $KE = MVC^2/2$, is gradually absorbed by the elastic and plastic deformations of the structure and its velocity decreases to a point where it comes to rest, then changes the direction of motion and finally detaches from the honeycomb at a velocity that is lower than its original strike velocity. The rigid plate was assumed to have frictionless contact with the honeycomb and periodic boundary conditions were imposed on the sides of the structure. Again, the out-of-plane degrees of freedom of the model were constrained to avoid the out-of-plane buckling.

Ruan et al. [52] showed that different deformation modes are observed for in-plane dynamic crushing of honeycombs in two different principal directions (ribbon or armchair vs. zigzag or transverse directions). However, note that the relative size of their sample was smaller than what we studied in the present case since we used periodic boundary conditions on the sides. It is indeed preferable to carry out comprehensive study on the influence of loading in either or both directions but in our current paper, we focus only on the zigzag direction especially since the other direction is assumed to be much larger. In other words, our study idealizes the core behavior of a long sandwich panel structure with a much larger dimension in the armchair direction which is under dynamic loads in the zigzag direction.

In all the simulations carried out in this study, the relative density was kept constant at 6%, unless otherwise stated.

3. In-plane dynamic crushing and impact behavior of regular hexagonal honeycombs

In this section, we investigated the effects of cell wall material strain hardening on the structural response of regular hexagonal...
honeycombs under constant-velocity dynamic crushing and impact loading using FE simulations.

3.1. In-plane dynamic crushing of regular hexagonal honeycombs

Fig. 1(A) shows the schematic diagram of the FE model of a regular hexagonal honeycomb structure undergoing in-plane dynamic crushing. Fig. 1(B) shows the plot of normalized plastic energy dissipation, \( \bar{U}_p \), versus the crushing strain, \( \varepsilon \), defined as \( \varepsilon = \delta / L \), where \( \delta \) is the vertical displacement and \( L \) is the initial height of the structure, for six different cell wall material strain hardening parameters, \( H \), and two different crushing velocities of the crushing rigid plate, \( V = 0.4 \) and \( V = 14 \). Here, \( \bar{U}_p = U_p / \sigma_{YC} AL \), where \( U_p \) is the plastic energy dissipation extracted directly from the FE results, \( \sigma_{YC} = 0.5 \rho c \sigma_Y \) is the effective yield stress for a regular hexagonal honeycomb [1], and \( A \) is the cross sectional area of the structure normal to the crushing direction. The figure shows that in the low velocity regime (\( V = 0.4 \)), the normalized plastic energy dissipation is significantly higher for higher values of \( H \) at a given crushing strain. In addition, we find that \( H \) also affects the rate of plastic energy dissipation with respect to the crushing strain. For instance, higher value of \( H \) corresponds to greater increase in normalized plastic energy dissipation for a given crushing strain increment. However, as the crushing velocity is increased, we find the difference in response due to variations in \( H \) is starkly reduced as seen in Fig. 1(B) (right) where the normalized plastic energy dissipation does not show much gain over the elastic–perfectly plastic envelope (\( H = 0 \)). In other words, higher crushing velocity seems to diminish the additional effects of strain hardening thereby bringing about a plasticity convergence.

To further investigate the plasticity convergence phenomenon, we studied the variation of plastic energy dissipation at different crushing velocities. Fig. 2(A) shows the normalized plastic energy dissipation as a function of the normalized crushing velocity at 50% crushing strain for the values of \( H \) ranging from 0 to 0.1. The results clearly show that as the crushing velocity increases, the normalized plastic energy dissipation for different values of hardening tangent modulus asymptotically approaches the normalized plastic energy dissipation of the elastic–perfectly plastic case. This asymptotic normalized plastic energy dissipation, \( \bar{U}_p(\bar{H} = 0) \), provides us with another convenient normalization constant which we use next to doubly normalize the normalized plastic energy dissipation. The plot of this doubly normalized plastic energy dissipation versus the normalized crushing velocity shown in Fig. 2(B) further demonstrates that increasing the crushing velocity diminishes the effects of material strain hardening. This confirms the plasticity convergence effect on the plastic energy dissipation at high crushing velocities. This diminishing influence of material strain hardening at high crushing velocities clearly points to broader underlying deformational differences at different crushing velocities.

These velocity induced changes in deformation patterns are probed using FE simulations on regular hexagonal honeycombs illustrated in Fig. 3. In the figure, we show the deformation modes of the honeycombs at different crushing velocities and cell wall material hardening tangent moduli at 50% crushing strain. From
the figure we observe three distinct deformation modes at three different crushing velocities: quasi-static or X-shape shear band, transition or V-shape shear band, and finally a localized dynamic mode characterized by a narrow crush band near the crushing plate. These deformation patterns are analogous to the localized deformations observed for regular hexagonal honeycombs with elastoplastic material behavior (in contrast to elastic material) under uniaxial compression [53,54]. For this case, Okumura et al. [53] showed that a long-wave buckling mode occurs in the structure. This long-wave buckling mode induces macroscopic localizations in the structure in the form of deformed cell rows analogous to localized deformations such as crush bands of different geometries we observed in our study. Interestingly, with increasing \( H \) at the lower crushing velocity \( (V = 0.4) \), the sharply defined X-shape (visible in the case of \( H = 0) \) loses its distinctive branches and the deformation increasingly diffuses throughout the structure. Similar phenomenon is seen for higher crushing velocity \( (V = 6) \), but with an initial V-shape shear band for the elastic–perfectly plastic case. However, for higher crushing velocity \( (V = 14) \), in spite of increasing the hardening tangent modulus, localized deformation in the form of a narrow crush band dominates the deformation mode confirming our earlier observation of plasticity convergence.

From these set of figures, one can further conclude that the general diffusive effect of increasing cell wall material strain hardening on deformation is somewhat arrested at higher crushing velocities. This behavior can be unified using a two dimensional deformation map as one shown in Fig. 4. The circles in this figure denote the points corresponding to deformation shapes of Fig. 3. The dashed lines represent the boundaries between two different deformation modes. The empirical equations for each of these dashed lines are also provided in Fig. 4, which have been estimated from the FE simulations. Results suggest that the critical normalized velocity at which transition from one deformation mode to another occurs, increases linearly with increasing cell wall material hardening tangent modulus. Thus, we can conclude from this map that increase in material strain hardening acts to delay the onset of the localized dynamic mode during dynamic crushing and the extent of the transition mode remains approximately constant across a range of crushing velocities and strain hardening parameter. The map also clearly elucidates the plasticity convergence phenomenon at higher velocities discovered in the earlier numerical simulations.

3.2. In-plane projectile impact of regular hexagonal honeycombs

In this subsection, we investigated the response of regular hexagonal honeycombs with linear cell wall material strain hardening under in-plane rigid plate impact. The schematic diagram of the FE model is shown in Fig. 5(A). We first investigate the effects of the cell wall hardening tangent modulus on crushing strain response. This is shown in Fig. 5(B) which plots the crushing strain of the honeycomb defined as \( \varepsilon = \delta / L \), where \( \delta \) is the vertical displacement and \( L \) is the initial height of the structure, versus the normalized time of crushing, \( t / t_0 \), where \( t \) is the actual time and
\[ t_0 = L/V_0. \] A relative density of 1% is taken for the simulations done in this part of the study. Normalized initial kinetic energy is defined as,

\[ KE = \frac{MV_0^2}{2} = \frac{\sigma Yc AL}{\sigma Yc + H^* \varepsilon_{max}}. \]

The term, \( \sigma Yc AL \), is an estimate for the total energy dissipated by plastic deformations of a bar with yielding stress, \( \sigma Yc \), length, \( L \), and cross-sectional area, \( A \) \[28\]. The results are presented for five different values of cell wall hardening tangent modulus with the normalized initial kinetic energy of \( KE = 0.58 \). Fig. 5(B) shows a strong dependence of maximum crushing strain on cell wall hardening tangent modulus where an increase in cell wall material strain hardening decreases the maximum crushing strain.

In order to derive an analytical estimate of the maximum crushing strain of the honeycomb, \( \varepsilon_{max} \), we assume that the initial kinetic energy of the rigid plate is completely dissipated by the plastic deformations of the honeycomb cell walls (i.e., perfectly inelastic collision) as it comes to rest. Using this assumption, \( MV_0^2/2 = (\sigma Yc + H^* \varepsilon_{max}/2)\varepsilon_{max} AL \), where \( H^* \) (as mentioned in Section 1) is the effective hardening tangent modulus of the structure. Note that the right hand side of this equation is simply the area under the stress–strain curve of the honeycomb multiplied by the volume of the structure (\( AL \)). This leads to the following relation:

\[ KE = \varepsilon_{max}(1 + H^* \varepsilon_{max}/2\sigma Yc) \]

The values of \( H^* \) are taken from the theoretical relation derived by Mangipudi et al. \[37\] which was provided in Section 1 of this paper. In addition, assuming the total initial momentum of the rigid plate to be equal to the impulse of the resisting force of the structure over the resting time of the plate, \( t_s \), we get an estimate for the resting time as, \( MV_0 = (\sigma Yc + H^* \varepsilon_{max}/2)At_s \). Dividing both sides of this equation by \( t_0 \) gives the relation, \( t_s/t_0 = 2KE/(1 + H^* \varepsilon_{max}/2\sigma Yc) \). Then, from this equation and Eq. (2), we obtain the following relation:

\[ t_s/t_0 = 2\varepsilon_{max} \]

It should be emphasized that the resting time, \( t_s \), also represents the time honeycomb takes to reach the maximum crushing strain. It is very clear that Eq. (2) indicates a nearly linear relationship between \( KE \) and \( \varepsilon_{max} \) when the hardening tangent modulus is small. This is confirmed by FE simulations shown in Fig. 5(C) (the FE results are shown by markers). This figure demonstrates that the higher value of \( H^* \) corresponds to a lower maximum crushing strain at a given initial kinetic energy of the impacting rigid plate.

The generally good agreement between FE and theoretical relationships derived in Eqs. (2) and (3) breaks down when considering in-plane impact response of regular hexagonal honeycombs made of a material with elastic-linear strain hardening property for 1% relative density. (A) Schematic of the finite element model. (B) The crushing response of honeycombs with different cell wall material hardening tangent moduli. (C) and (D) The maximum crushing strain and the normalized crushing time versus the normalized initial kinetic energy of the impacting rigid plate. The markers show the results of finite element simulations and the solid lines represent the results of theoretical estimations.
somewhat at higher kinetic energies (see Fig. 5(C) and (D)) mainly due to the dynamic effects and nonlinearities caused by cell wall contacts [28] in the high kinetic energy regime.

Interestingly, in contrast to the constant-velocity dynamic crushing response observed in the previous subsection, no plasticity convergence effect is seen in the impact behavior of the honeycombs as higher KE does not in general lead to diminishing the differences between the structures with different material strain hardenings.

4. In-plane dynamic crushing and impact behavior of functionally graded hexagonal honeycombs

In this section, we studied the in-plane dynamic crushing and impact response of hexagonal honeycombs with FG relative density introduced by systematic variations of cell wall thickness in the direction of crushing. The gradient scheme is illustrated in Fig. 6(A), where we divided the structure into five equal-size substructures of length $\Delta l$, where $\Delta l = l/5$. The density gradient is then defined as $\psi = (\rho_i + 1 - \rho_i)/\Delta l$, where $\rho_i$ is the relative density of the structure in the ith sub-structure (see Fig. 6(A)) [28]. In this study, the overall relative density of the structure was kept constant at 6%. Therefore, $\psi = 0$ represents a regular hexagonal honeycomb with uniform relative density and $\psi > 0$ denotes a FG honeycomb in which the relative density is reduced in the direction of crushing although the overall relative density is still 6%. To sum, a positive density gradient entails moving material from the bottom layers and adding it to the top layers keeping the overall mass (density) constant.

4.1. In-plane dynamic crushing of functionally graded hexagonal honeycombs

Figs. 6(B) and 6(C) respectively show the normalized plastic energy dissipation of FG hexagonal honeycombs up to 50% crushing strain for five different density gradients at low and high velocity regimes of in-plane dynamic crushing when the cell wall material has an elastic-perfectly plastic property ($\bar{\rho} = 0$). Two pairs of density gradients are imposed with each pair consisting of density gradation of the same magnitude but in opposite direction and compared with a regular hexagonal honeycomb (no density gradient, $\psi = 0$). In the lower velocity regime plotted in Fig. 6(B), the effect of introducing density gradient in either direction degrades the energy absorption capacity of the honeycomb. This can be attributed primarily to the weakening of the overall structure due to decrease in cell wall thickness in some members brought about by density gradient in any direction. This adverse effect of weakened members on plastic energy dissipation increases with increasing density gradient due to even greater lowering of members’ strength. However, when the crushing velocity is increased substantially, only the layers closer to the crushing plate are initially engaged. Thus, the significant stiffening of members due to positive density gradient causes higher plastic energy dissipation for a given crushing strain in the initial stages of crushing while the opposite being true for negative density gradient. Indeed such a trend is clearly observed in Fig. 6(C) which shows that more plastic energy dissipation occurs for positive density gradient and less for negative density gradient compared to a regular hexagonal honeycomb of the same total mass in the higher crushing velocity regime. We expect similar trends when cell wall material strain hardening is introduced in both the low and high velocity crushing simulations shown respectively in Figs. 6(D) and 6(E) for 10% strain hardening ($\bar{\rho} = 0.1$). Thus, unlike the velocity induced plasticity convergence observed in Section 3.1, here we do not find a similar gradient convergence (i.e. diminishing the effects of FG on crushing response at higher crushing velocities). However, comparing Fig. 6(C) with 6(E), we find that the plasticity convergence phenomenon is still present, since the introduction of material strain hardening for the higher crushing velocity has little effect on plastic energy dissipation of the honeycombs with the same density gradient as opposed to significant differences in the lower velocity regime as seen by comparing Fig. 6(B) with 6(D).

In the discussion above, we have postulated a mechanism explaining the mechanical response of FG hexagonal honeycombs based on deformation shapes. In order to shed more light on the influence of density gradient and cell wall material strain hardening on deformation of these honeycombs, we compare the deformation modes of FG honeycombs to a constant density honeycomb ($\psi = 0$) for two different cell wall material hardening tangent moduli. Fig. 7(A) and (B) show the deformation modes of FG honeycombs for two different values of density gradient, $\psi = 0$ and $\psi = +0.17$, two different hardening tangent moduli of the cell wall material, $\bar{\rho} = 0$ and $\bar{\rho} = 0.1$, at 50% crushing strain for both low velocity, $V = 0.4$, as well as high velocity, $V = 14$, dynamic crushing, respectively. For regular honeycombs ($\psi = 0$), the effects of crushing velocity as well as cell wall material strain hardening on deformation modes have already been investigated in detail in Section 3.1 of this paper (see Fig. (3)). For the low velocity regime shown in Fig. 7(A), we find that for both $\bar{\rho} = 0$ and $\bar{\rho} = 0.1$, introducing density gradient causes the deformation to be localized at the part of the structure which has the lowest relative density and thus stiffness (i.e. the bottom part of a FG honeycomb with positive density gradient) instead of localized X-shape shear bands seen in their regular counterparts. As discussed above, this key difference of collapse of the weaker members during low velocity crushing regime causes higher crushing strains but lower plastic energy dissipation due to reduced thickness of the collapsing members. Moreover, for Fig. 7(A) (right) where cell wall material hardening is introduced, the preferential collapse of the weaker members relocates the uniformly diffusive effect of cell wall material strain hardening on deformation which is otherwise seen in their regular counterpart near the weaker members of the structure. On the other hand, when the crushing velocity is increased (see Fig. 7(B)), only the part near the crushing plate is engaged in deformation as a localized crush band form. Thus, plastic energy dissipation is strongly dependent on the properties of the members in this region. Hence, for a positive density gradient honeycomb, a higher dissipation is observed whereas the reverse is true for a negative density gradient honeycomb. The effect of cell wall material strain hardening is not seen to reverse this phenomenon. This confirms our explanations for Fig. 6(C) and (E).

4.2. In-plane projectile impact of functionally graded hexagonal honeycombs

In this subsection, we investigated the effect of density gradient as well as cell wall material strain hardening on impact response of FG hexagonal honeycombs. The schematic diagram of the FE model is shown in Fig. 8(A). Figs. 8(B) and 8(C) show the maximum crushing strain, $\varepsilon_{\text{max}}$, of FG honeycombs with different density gradients versus the normalized initial kinetic energy, $KE$, of the impacting rigid plate for two different cell wall material hardening tangent moduli, $\bar{\rho} = 0$ and $\bar{\rho} = 0.1$, respectively. From the figures we conclude that introduction of density gradient in both directions increases the maximum crushing strain at a given initial kinetic energy up to a fairly high crushing strain, though the difference steadily decreases with increasing kinetic energy and the results converge at 60% maximum crushing strain. These trends can be explained by recalling the discussion on constant-velocity dynamic crushing in the previous subsection. Thus, the behavior observed at lower KE, similar to the low velocity dynamic crushing of FG honeycombs, is primarily due to the weaker
members absorbing lesser amount of elastic and plastic energies as well as collapsing earlier than regular members. However, as the impacting energy increases, the crushing of even regular members can take place thereby diminishing the difference between the regular and FG honeycombs up until convergence occurs at 60% maximum crushing strain. This convergence of maximum crushing strain however occurs at a greater value of the normalized initial kinetic energy for the case of 10% cell wall material strain hardening compared to the elastic–perfectly plastic case.

The difference in the values of the maximum crushing strain upon introduction of density gradient in both directions gradually ends as the kinetic energy is increased thereby exhibiting gradient convergence analogous to the plasticity convergence phenomenon observed earlier. This change in the impact behavior towards convergence is primarily due to the localization of deformations directly underneath the impacting plate at higher velocities demonstrated earlier in Fig. 7(B). Thus, we can idealize the initial period of a higher velocity impact as a perfectly inelastic collision resulting in the motion of a crushed band of material directly underneath the impacting plate through the honeycomb structure. Since the velocity in this period can be approximated by a momentum drop, greater crush band mass results in greater velocity drop. Thus, we would expect that there would be greater decrease in velocity for positive FG and lesser drop for negative FG compared to regular honeycombs in this period.

Fig. 6. In-plane dynamic crushing of functionally graded hexagonal honeycombs. (A) Schematic of the finite element model. (B) Normalized plastic energy dissipation versus the crushing strain for honeycombs with different density gradients at low crushing velocity, $V = 0.4$ and $H = 0$ and (C) at high crushing velocity, $V = 14$ and $H = 0$ and (D) $V = 0.4$ and $H = 0.1$ and (E) $V = 14$ and $H = 0.1$. 

$$\psi = \frac{\rho_{i} - \rho_{0}}{\Delta L} \text{ for } i = 1 \text{ to } 4$$
To confirm this claim, we present the plots of crushing strain rate, \( \dot{\varepsilon} \), and crushing strain, \( \varepsilon \), versus the normalized time of impact for the honeycombs with 10% cell wall material hardening tangent moduli, \( H = 0 \) and \( H = 0.1 \) at 50% crushing strain at (A) low crushing velocity, \( V = 0.4 \) and (B) at high crushing velocity, \( V = 14 \).

Fig. 7. Deformation modes of functionally graded honeycombs for two different density gradients, \( \psi = 0 \) and \( \psi = +0.17 \) and for two different cell wall material hardening tangent moduli, \( H = 0 \) and \( H = 0.1 \) at 50% crushing strain at (A) low crushing velocity, \( V = 0.4 \) and (B) at high crushing velocity, \( V = 14 \).

5. Concluding remarks

In this paper, numerical models were employed to study the effects of cell wall material strain hardening on the structural response of regular and FG hexagonal honeycombs under in-plane dynamic crushing and impact loading. Note that although traction free boundary conditions [52] seem reasonable for the current study, we chose to employ periodic boundary conditions for the left and right sides of the structure, which are typically used for representative volume element (RVE) simulations [55]. Such boundary conditions have already been used in similar situations in earlier studies [28,51,56,57]. For the current study, this kind of boundary condition eliminates boundary effects since it assumes the structure to be replicated infinitely in the horizontal direction. Therefore, these boundary conditions would be less accurate for the case where the sample size is of the order of the RVE or if the overall structure is morphologically dissimilar to the RVE chosen in the current problem [55].

For dynamic crushing of regular honeycombs, we found that cell wall material strain hardening has a general diffusive effect on the honeycomb deformation patterns which acts to mitigate the localization effect of greater crushing velocities. This was illustrated using a unified phase map indicating distinct quasi-static, transition, and dynamic modes separated by velocity and material strain hardening dependent phase boundaries. These simulations also showed overwhelming deformation localization at higher crushing velocities mitigated the diffusive effect of material strain hardening on the crushing phenomena thereby exhibiting a phenomenon called plasticity convergence. In addition to dynamic crushing, we also investigated the effect of dynamic impact on honeycombs using numerical and analytical modeling. The analytical analysis indicated a nearly linear relationship between initial kinetic energy of the impacting plate and maximum crushing strain of the structure as well as resting time of the plate at low hardening tangent moduli of the cell wall material. We discovered that increasing the material strain hardening decreases the rate of increase in maximum crushing strain as well as resting time with initial kinetic energy of impact. These results were verified using FE analysis.

Next, we studied the influence of density functional gradation in conjunction with material strain hardening on the dynamic crushing and impact response of hexagonal honeycombs. We found that FG itself significantly alters the response of the honeycomb compared with their regular counterparts. A preferential collapse of the weaker (hence thinner) members during lower crushing velocities thus diminished the plastic energy dissipation for a given crushing strain. On the other hand, higher crushing velocities brought about deformation localization similar to the one observed for regular honeycomb and thus at a given crushing strain, more positive FG indicating stronger members directly underneath the crushing plate showed greater plastic energy dissipation. Interestingly, material strain hardening which retained its diffusive effect observed earlier for regular honeycombs, also brought about significant differences in plastic energy dissipation at lower crushing velocities. However, localization of failure underneath the crushing plate once again brought about plasticity convergence between corresponding FG honeycombs at high crushing velocities. Note that unlike plasticity convergence, we did not observe any analogous gradient convergence (i.e. convergence of behavior across FG due to higher crushing velocity) at higher crushing velocities. However, an analogous gradient convergence was indeed observed in the case of impact with respect to maximum crushing strain at higher kinetic energy of impact. Although, this was found to be primarily caused by the localization of deformation during the initial phases of impact, the exact mechanism was found to differ from plasticity convergence. Since material strain hardening in general works at cross purpose to localization, its effect was seen in requiring higher threshold of kinetic energy for this gradient convergence to occur.

To sum, our study serves an important step in both furthering our understanding of the critical role of plasticity of the cell wall material of honeycombs under dynamic loadings as well as providing a guideline to design advanced systems which can...
incorporate these honeycombs in the vicinity of the operating conditions described here.

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