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Contact kinematics of biomimetic scales

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Dermal scales, prevalent across biological groups, considerably boost survival by providing multifunctional advantages. Here, we investigate the nonlinear mechanical effects of biomimetic scale like attachments on the behavior of an elastic substrate brought about by the contact interaction of scales in pure bending. We carried out a computational study of more modern, thinner, and more compliant teleost scales showing significant strain-stiffening effects due to scale bending. Our results reveal the existence of three distinct kinematic phases of operation spanning linear, nonlinear, and rigid behavior driven by kinematic interactions of scales. The response of the modified elastic beam strongly depends on the size and spatial overlap of rigid scales. The nonlinearity is perceptible even in relatively small strain regime and without invoking material level complexities of either the scales or the substrate. © 2014 AIP Publishing LLC.

Scale like dermal modifications have evolved for millions of years in diverse biological species and are ubiquitous in nature due to their multifunctional and protective roles.1–6 Fig. 1. For instance, scales may aid in coloration and camouflage,7–12 hydrostatic balance,13 sensing and locomotion,14–15 or as direct mechanical protection against predators.6,16,17 Interestingly, altering material, spacing and geometry of scales can yield both mechanical protection and simple optical functions such as reflection, cloaking, and camouflage.18 However, more complex, chromatically functional scales tend to be much smaller (e.g., 4–6 μm for Papilo palinurus19) and often contain further in-scale morphological features such as concavities,19 stripes,20 and longitudinal ridges and cross-ribs21 for greater influence on light reflection and polarization. Scales of fishes which must balance protection against powerful predators with high degree of agility due the aquatic nature of their locomotion,22,23 have come under increasing scrutiny for advancing armor research with the primary focus on extracting insights from the material constitution. For instance, the remarkable penetration resistance of the dermal armor of a typical gray bichir (Polypterus senegalus) was found to be primarily due to the layered, graded material constitution, the management of micro crack evolution and load influenced material property variation of its ganoid scales.16 Tensile tests and fractography of Arapaima gigas showed characteristics of typical laminated nano-composite for both dry and wet scales.24 Further investigation using in-situ synchrotron small-angle X-Ray scattering on A. gigas showed a Bouligand-type structure allowing the lamellae to reorient in response to the loading environment enhancing their protective properties.25 A computational study of more modern, thinner, and more compliant teleost scales showed significant strain-stiffening effects due to scale bending.26 Perforation experiments carried out on Morone saxatilis (striped bass) showed a two stage penetration as the cause of penetration resistance.27

The general mechanical principles for adopting scales for dermal armors were soon being explored.17 This included manufacturing an elementary composite system based on the geometry of fish scales28 and a more general dual-track additive manufacturing combining computational modeling and multi-material 3-D printing to fabricate bio-inspired, multi-scale, and tunable composites.29,30

In this letter, we investigate the influence of biomimetic scale like modification on the behavior of an elastic system at a structural/system level.

We deliberately simplify the complex material response of scales and substrates to isolate the nature of nonlinearity arising purely from the interaction of scales at the system level which is distinct from the already extensively investigated material sources. Figs. 2(a) and 2(b) show the bending response of a simple prismatic beam with biomimetic scales. To prepare the samples, first, we fabricated a mold for the substrate using a 3D printing machine (Objet, Eden333) with Acrylonitrile butadiene styrene (ABS) thermo-polymer. The mold was filled with a liquid silicon based elastomer, Vinylpolysiloxane (VPS), with elastic modulus 1.4 MPa. The scales were also printed using 3D printing machine from ABS thermo-polymer (elastic modulus 2.3 GPa) and were then glued to the VPS substrate using 3M plastic adhesive (3M Corporate, St. Paul, MN). The elastic modulus of the ABS thermo-polymer was roughly 2.3 GPa. Fig. 2(a) describes the two different bending configurations of the resulting biomimetic structure, the upper one bent into a convex curvature with no scale engagement and the lower one into a concave curvature with significant scale engagement. Fig. 2(b) shows the bending response of two specimens—beam with no scales and beam with biomimetic scales measured using an Instron universal testing machine with 1 kN load cell. From this figure, it is clear that there was a significant enhancement in bending resistance of the beam with biomimetic scales. It should be noted that the presence of scales increased even the initial stiffness of the biomimetic structure when compared with the virgin beam indicating an intrinsic increment in stiffness even before scale engagement. Motivated by our qualitative experimental results, we now develop a theory to predict the contact kinematics and interaction behavior of surface scales and their role in...
altering their pure bending behavior. We assumed a linear elastic substrate and the scales were modeled as rigid, frictionless links embedded on the top surface of the substrate, Fig. 3(a). The difference in elastic modulus between a typical scale (\(\sim\)GPa)\(^{17,27}\) and skin (\(\sim\)MPa)\(^{31}\) is large enough to warrant the rigidity assumption.

Beyond these material assumptions, we assumed that the scales interaction preserves the periodicity of self-contacts. This assumption is strictly valid only for infinitely long beams under pure bending. For the scales, we denoted the length of the embedded part by \(L\) and the length of the exposed part by \(l\) such that the total scale length \(l_s = L + l\).

FIG. 2. (a) A manual illustration of the system deformation under bending in two opposite directions. (b) Comparative force-displacement plots for a three point bending experiment conducted on the fabricated sample using an Instron universal testing machine with displacement control to show effect of scale engagement on bending behavior of beam. Experiments were repeated 4 times on the same sample and the bars show standard deviation of experimental results. Fabricated samples dimensions 200 mm (length) \(\times\) 20 mm (width) \(\times\) 10 mm (height) and scales dimensions were 35 mm \(\times\) 20 mm \(\times\) 1 mm with spacing of 10 mm and angle of \(10^\circ\) measured from the substrate surface. In the experiment, span of supports for 3-point bending was 100 mm and the machine pushed the beam at constant speed of 10 mm/min.
We further assume that scale thickness \( D \ll l \), and the beam height \( h \gg L \). These slender plate-like scales are a good geometric idealization of many types of scales found in nature.\(^{28}\)

In addition to these scale parameters, the inter-scale distance (scale separation) denoted by \( d \) is also an important geometrical parameter which determines both areal density as well as the scale overlap of the structure. Specifically, we define \( \eta = l / d \) as the overlap ratio. Note that although none of the above parameters may be constant across scales in real organisms non-dimensionalization imposes considerable limitations on the effect of individual variability and highlights some of the most salient features of this system. The bending of the underlying beam (assumed Euler-Bernoulli) causes the scales to rotate from their initial angle of \( \Theta_0 \) with respect to the beam centerline to angle \( \theta \). The periodicity assumption leads to a geometrical configuration shown in Fig. 3(a) indicating the existence of a fundamental repeating structural unit called the representative volume element (RVE). From the geometry of the RVE shown in Fig. 3(a), we can derive the following nonlinear geometric relationship.\(^{72}\)

\[
\frac{\eta \psi \sin \theta - (1 - \cos \psi)}{\eta \psi \cos \theta - \sin \psi} - \tan(\theta + \psi) = 0, \tag{1}
\]

where \( \psi \) is the angular rotation of each RVE with respect to the instantaneous center of curvature of the beam. The RVE base rotation \( \psi \) is directly related to instantaneous curvature \( \kappa \) of the substrate as \( \psi = \kappa d \). Note that Eq. (1) represents the geometric nonlinearity inherent in the problem due to the contacting scales. This equation can, however, be linearized as \( \theta = (\eta - 1/2) \kappa d \) for \( \psi, \theta \ll 1 \). From this equation, the scale engagement curvature \( \kappa_E = \frac{1}{2} \frac{\partial \Theta}{\partial \theta} \). The kinematics governed by the nonlinear equation (1) can be used to construct a map spanned by the two geometric parameters—the scale angle and substrate rotation angle, Fig. 3(b) for \( \Theta_0 = 45^\circ \). The map indicates that the performance locus of this biomimetic system spans three distinct kinematic regimes of operations. When the substrate rotation is sufficiently small or overlap ratio less than a critical value \( \eta_c = \sin \Theta_0 / \Theta_0 \ll 1 \) (Ref. 72), there is no scale engagement and the system behaves as an elastica thereby remaining on the linear boundary of the phase map. As the beam curvature increases, engagement starts at a finite curvature and the scale rotation thereafter proceeds according to the relation given by Eq. (1). The scales of different geometries carve out a space mapped through the overlap ratios denoting a nonlinear response region. In this region, the scales continually slide against each other till they finally arrive into a kinematically locked configuration corresponding to the rigid phase boundary.

Locking is achieved when the scales have slid sufficiently till a maximum curvature is reached. This maximum possible substrate rotation \( \psi_{lock} \) is obtained by solving the transcendental equation \( \psi_{lock} \cos \psi_{lock}/2 = 1/\eta \) and the locking locus shown in Fig. 3 can be shown to be given by the equation \( \psi_{lock} + \psi_{lock}/2 = \pi/2 \) where \( \psi_{lock} \) is the locking scale inclination angle.\(^{72}\) Thus, if \( \Theta_0 > \Theta_{lock} \), then post-engagement, the scale inclination will decrease with increasing beam curvature till locking is achieved.

It is clear from the previous discussion that the entire sequence of bending can be envisaged as a combination of beam bending and scale rotation. The scale rotation is resisted by the underlying beam which in the small deformation regime can be assumed to act like a linear torsional spring. Thus if the current scale angle is given by \( \theta \), the energy \( U_{scale} \) absorbed by a single scale rotation per unit depth of the beam would be \( U_{scale} = 1/2 K_B (\Theta - \Theta_0)^2 \), where \( K_B \) is the torsional spring constant. The important variables that determine \( K_B \) are quite evidently the Young’s modulus of the base \( E_B, D, L \), and \( \Theta_0 \). Thus, the following non-dimensional relationship emerges under the assumption \( 0 \ll L \ll h \).\(^{72}\)

\[
\frac{K_B}{E_B D^2} = C_B \left( \frac{L}{D} \right)^n f(\Theta_0), \tag{2}
\]

where \( C_B \) and \( n \) are dimensionless constants and \( f(\Theta_0) \) is a dimensionless angular function to track initial scale orientation effects. Since the effect of Poisson’s ratio variation was not considered in this study, its contribution is deemed absorbed into \( C_B \). To calculate the values of the dimensionless constants, we carried out FE simulations on a system consisting of a single scale with varying geometry and...
inclination, embedded in a large rectangular domain to simulate an infinite media, Fig. 4(a). In the simulations, the size of the rectangular domain representing the substrate was increased till no change in the scale response was perceived. The results show an excellent agreement with Eq. (2) with dimensionless \( C_B = 0.66 \), \( n = 1.75 \), and \( f(\theta_0) \approx 1 \) indicating almost no angular dependence of the base rotational stiffness. Next, we equate work and energy at the RVE level

\[
\int_{0}^{\kappa} M(\kappa')d\kappa' = \frac{1}{2} E_B I\kappa^2 + \frac{1}{2} K_B (\theta - \theta_0) \frac{1}{d} H(\kappa - \kappa_c). \tag{3}
\]

Here \( M \) is the applied moment, \( I \) is the second moment of area of the beam cross-section, and \( H(.) \) is the Heaviside step function. Differentiating both sides of Eq. (3) with respect to \( \kappa \) yields the moment-curvature relationship

\[
M = \frac{E_B I \kappa}{d} + \frac{K_B (\theta - \theta_0)}{d} \frac{\partial}{\partial \kappa} H(\kappa - \kappa_c). \tag{4}
\]

Interestingly, this relationship which predicts identical stiffness for the biomimetic and virgin beam till the point of engagement seems to contradict the difference in initial stiffness observed experimentally in Fig. 2(b) and mentioned earlier. However this apparent contradiction is merely due to the fact that our current analytical model did not consider the contribution of an elastic boundary layer determined by the finite width of the scale-beam embedded region which tends to constrict material flow and which endows an intrinsic stiffness to the beam even prior to engagement. Now assuming small deformation and plugging the expression of \( K_B \) into Eq. (4), we get

\[
M = \frac{E_B I \kappa}{d} + \frac{K_B (\theta - \theta_0)}{d} \left( \frac{L}{D} \right)^n \left( \eta - \frac{1}{2} \right) (\kappa - \kappa_c) H(\kappa - \kappa_c). \tag{5}
\]

We compared the above constitutive model with a FE simulation of a similar system in pure bending using, Fig. 4(b) where \( \theta_0 = 10^\circ \) in order to highlight the emergence of nonlinear behavior even at a small curvature. In the FE models, rigid body constraint as well as surface to surface frictionless contact was imposed on the scales and sufficient mesh density was employed to achieve convergence. The differences between the FE and the theoretical formula arise mainly due to, first, edge effects since an infinitely long substrate is not physically possible and secondly, the neglect of the elastic boundary layer in the analytical model. From the above analysis, it is clear that the gain in stiffness owes directly to scale engagement (and to some extent the boundary layer constriction). Therefore, this nonlinear effect shall not be limited to pure bending load and may exhibit in other types of complicated loads such as indentation. Equation (5) predicts a quadratic relation of the stiffening term on the overlap ratio \( \eta \), which increases due to either an increase of scale length or decreasing scale spacing \( d \). Note that a lower \( d \) also contributes to a declining engagement curvature, a direct conclusion from the engagement curvature expression. At the same time, note that the initial scale inclination angle \( \theta_0 \) determines the span of the nonlinear region affecting the engagement curvature. However, very long, spiny, and high inclination scales may impede the overall mobility and maneuverability and in addition may increase scale compliance thereby mitigating the stiffness gains. These factors seem to contribute towards putting a limit on the overall length and inclination of the scales with respect to the organism size found naturally. These deductions are independent of the material properties and are by and large structural features.

To conclude, our study highlights that the most interesting feature of this seemingly simple system is the origin of the nonlinearities arising purely out of self-contact in the small strain regime. This relatively simple mechanism for engendering nonlinearity conforms to an important recurring theme of biological systems which are often constrained by material choices but manage to exhibit rich mechanical behavior using different evolutionary adaptations such as hierarchy, chirality, self-assembly, nano-confinement, nano-structural transitions, and novel topological deployment.