Mechanical behavior and failure of composite pyramidal truss core sandwich columns

Jian Xiong\textsuperscript{a,b}, Li Ma\textsuperscript{a}, Linzhi Wu\textsuperscript{a,*}, Jiayi Liu\textsuperscript{a}, Ashkan Vaziri\textsuperscript{b}

\textsuperscript{a}Center for Composite Materials and Structures, Harbin Institute of Technology, Harbin 150001, PR China
\textsuperscript{b}Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA 02115, USA

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\textbf{A B S T R A C T}

A series of analytical and experimental investigations is presented to study the response and failure of pyramidal truss core sandwich panels made of carbon fiber composite under axial compression. In the analytical part of the study, three failure modes: (i) Euler or core shear macro-buckling, (ii) face wrinkling, and (iii) face sheet crushing, were considered and theoretical relationships for predicting the failure load associated with each mode were presented. In the experimental part of the study, three different sets of specimens were manufactured to probe the three failure modes mentioned above. Pyramidal truss cores were fabricated using a hot-press molding technique and were bonded to composite face sheets made of fiber reinforced composite. The response of the sandwich panels under axial compression was measured up to failure. The measured peak loads obtained in the experiments showed good agreement with the analytical predictions. The experiments also provide insight into the post-failure response of the sandwich panels.

\textsuperscript{*}Corresponding author. Tel.: +86 451 86412549; fax: +86 451 86402386.
E-mail addresses: xiongjian0309@126.com (J. Xiong), wlz@hit.edu.cn (L. Wu).

1. Introduction

Sandwich panels with low density cores are traditionally manufactured using stochastic foams and lattice materials such as hexagonal honeycombs [1–3]. The emergence of new manufacturing techniques for building three-dimensional periodic cores (e.g. pyramidal and Kagome cores, X-cor trusses) has opened new opportunities for designing lightweight multifunctional structures [4–11]. An inherent part of this progress is the need to understand the mechanical behavior of these novel structures. In this context, the available literature is mainly focused on the behavior of metal sandwich panels, due to their potential applications in building large scale structures and ships, as well as energy absorbent and protective components. Use of fiber reinforced composites in sandwich structures generally allows an additional weight reduction without jeopardizing the strength and performance of the structure. Thus, sandwich panels made of fiber reinforced composites are attractive for building ultra-light, high strength components, specifically for the aerospace industry and for flight structures [12–20].

Compared to metal sandwich panels, studying the behavior of composite sandwich panels is more challenging due to the complex, and generally highly anisotropic, behavior of the composite. Most of the studies on the mechanical response of composite panels are focused on investigating the core crushing behavior and out-of-plane compressive response of the panels [12,14–18]. In the present work, we studied the response of carbon fiber composite pyramidal core sandwich panels subjected to axial compression using analytical models and experiments. Fig. 1a shows the schematic of a pyramidal sandwich panel with length \(L\), face sheet thickness, \(h_f\), and core height, \(h\), subjected to in-plane quasi-static compression. Fig. 1b shows the schematic of a unit cell of a pyramidal truss core with thickness, \(t\), width, \(d\) and height, \(h\). The density of the truss core, \(\rho\), is related to the core material density, \(\rho_s\), by:

\[ \rho = \frac{(2b \cos \omega + h - 2t)dt}{(h + 2bt)^2 \cos^3 \omega} \rho_s \]  

where \(b\) and \(\omega\) are geometrical parameters shown in Fig. 1b. The relative density of the core is \(\rho = \rho/\rho_s\).

In Section 2, we derive analytical expressions for the failure load of composite sandwich columns under in-plane compression by expanding the previous analytical work on metal sandwich columns with pyramidal truss cores [21] and sandwich columns made of glass fiber reinforced epoxy face sheets with PVC polymer foam cores [22,23].

In the experimental part of the study, we first manufactured carbon fiber composite pyramidal truss cores using the hot-press molding technique described in our recent published work [18]. In this method, all the continuous fibers of the composite are aligned in the direction of struts and thus the truss structure can...
fully exploit the intrinsic strength of the fiber reinforced composite. The fabricated truss structures were shown to have strength close to the theoretical limit of lattice structures [18]. For manufacturing the carbon fiber composite pyramidal truss cores, we used unidirectional carbon fiber/epoxy prepreg sheets with a thickness of 0.15 mm (T700/epoxy composite, Beijing Institute of Aeronautical Materials, China). The properties of a unidirectional prepreg are listed in Table 1. The fabricated composite truss structures were employed as the core material for manufacturing composite sandwich panels. Details about the materials and manufacturing process of the sandwich panels are presented in Section 3. The experimental results and the comparison with the analytical predictions are discussed in Section 4. The conclusions are drawn in Section 5.

2. Analytical predictions for competing failure modes

Three different failure modes of sandwich panels under axial compression, shown schematically in Fig. 2, were considered: (i) Euler or core shear macro-buckling, (ii) face wrinkling, and (iii) face crushing including face delamination and plastic microbuckling. Cote et al. [21] studied the first two failure modes listed above for metal sandwich columns with a pyramidal truss core and provided analytical estimates of the failure force for each mode. In the following sections, we will present similar analytical predications for the failure modes of a pyramidal core sandwich column made of carbon fiber reinforced composite.

2.1. Euler and core shear macro-buckling

Euler buckling and core shear buckling are two possible modes of elastic macro-buckling in a sandwich panel under axial compression (i.e. buckling modes with wavelength in the order of panel dimensions). The Euler buckling load, \( P_{\text{E}} \), can be estimated from classical buckling theory [24] as:

\[
P_{\text{E}} = \frac{k^2 \pi^2 (E)_{eq}}{L^2}
\]

where \((E)_{eq}\) is the equivalent flexural rigidity of the composite column and \(k = 2\) for a column with built-in ends. Assuming that the Poisson’s ratios of the core in the in-plane directions are negligible, the equivalent flexural rigidity of the composite column can be estimated from classical laminate composite beam theory [25].

In studying the core shear buckling, it is reasonable to neglect the shear stiffness of the face sheets and assume that the shear rigidity of the sandwich column is approximately equal to that of the pyramidal core. The core shear buckling load, \( P_{\text{S}} \), can be estimated from classical buckling theory [24] as:

\[
P_{\text{S}} = \frac{\left(2b \cos \omega + h - 2t\right) \pi^2}{8(h + 2b)^2 \cos^3 \omega} E_v
\]

If the two buckling loads associated with the sandwich panel macro-buckling, \( P_{\text{E}} \) and \( P_{\text{S}} \), are considerably different, then Eqs. (2) and (3) can be used to estimate the critical buckling load, \( P_{\text{c}} \), (i.e. If \( P_{\text{E}} \gg P_{\text{S}} \), then \( P_{\text{c}} \approx P_{\text{E}} \), if \( P_{\text{S}} \gg P_{\text{E}} \), then \( P_{\text{c}} \approx P_{\text{S}} \)). However, if the Euler and core shear buckling loads are of the same order of magnitude, the interaction between buckling modes should be considered. In this case, the critical buckling load can be estimated from, \( 1/P_{\text{c}} = 1/P_{\text{E}} + 1/P_{\text{S}} \).

2.2. Face wrinkling

The face sheet of a sandwich panel is generally much stiffer than its low density core, thus, the strain mismatch between the face sheet and core (here, induced by axial compression) could lead to face sheet instability in the form of wrinkles with a short wavelength [27,28]. In our study, we assumed that the face sheets behave elastically and the wavelength of the face wrinkles is equal to the distance between the core attachment points to the face.

| Table 1 Properties of unidirectional laminate (T700/epoxy composites). |
|-------------------|------------------|
| Properties        | Value            |
| 0° Tensile strength (MPa) | 1400             |
| 0° Tensile modulus (GPa)  | 123              |
| 90° Tensile strength (MPa) | 18              |
| 90° Tensile modulus (GPa)  | 8.3              |
| 0° Compression strength (MPa) | 850             |
| 0° Compression modulus (GPa)  | 100             |
| 90° Compression strength (MPa) | 96              |
| 90° Compression modulus (GPa)  | 8.4              |
| In-plane shear strength (MPa)  | 16.0             |
| In-plane shear modulus (GPa)  | 4.8              |
| Interlayer shear strength (Mpa) | 60             |
| Poisson’s ratio          | 0.3              |
| Volume fraction of fibers | 57% ± 3          |
| Density (kg/m³)          | 1550             |

Fig. 1. (a) Schematic diagram of the compression test assembly. (b) Sketch of the unit cell of the pyramidal core.
sheet, denoted by \( l \) in Fig. 1a, where \( l = (2h - 4t + 4b) \cos \omega \). The force associated with the elastic wrinkling of the face sheet, \( P_{W} \), can be estimated from
\[
P_{W} = \frac{2\pi^{2}(EI)}{l^{4}}
\]
where \((EI)\) is the flexural rigidity of the face sheet in respect to the panel mid-plane, which can be obtained using classical laminate composite beam theory [25].

2.3. Face sheet crushing

Since the pyramidal truss core is much more compliant compared to the face sheets, the face sheet crushing load can be estimated from,
\[
P_{Y} = 2\sigma_{0}h_{y}w
\]
where \( \sigma_{0} \) is the microbuckling strength of the composite face sheet, which depends on the degree of fiber misalignment and the matrix shear strength (see for example the review by Fleck [22,29]). In the present study, we calculated \( \sigma_{0} \) using the composite laminate theory [25] and Hashin criteria with a sudden degradation model [30–32] using the ANSYS Parametric Design Language (APDL). The Hashin criteria in the three-dimensional form are given in Table 2.

3. Manufacturing of sandwich panels

Fig. 3 shows the schematic of the mold used for fabrication of the cores. The mold consists of four different parts: (1) up web frames, (2) down web frames, (3) blocks and (4) the base tooling, which was made of chrome steel. The expansion blocks used in the process are cast silicon rubber and were laid into the space between the base tooling and the down web frames. Fig. 4a shows an example of the manufactured composite pyramidal lattice structures, where each strut of the pyramidal truss is made of six slender laps and has a thickness, \( t = 0.9 \) mm. The pyramidal truss structure shown in Fig. 4a has \( b = 4 \) mm, \( h = 15 \) mm, \( d = 3 \) mm, \( \omega = 45^\circ \), and relative density \( \rho = 1.81\% \). The sandwich panels were fabricated by bonding the pyramidal truss core to flat carbon fiber reinforced face sheets with an adhesive (08–57, Heilongjiang Institute of Petrochemical Industry). Fig. 4b shows an example of the sandwich panel with a pyramidal truss core with \( b = 4 \) mm, \( h = 15 \) mm, \( d = 3 \) mm, \( \omega = 45^\circ \), \( \rho = 1.25\% \).

Three sets of sandwich panels with different face sheet thickness, \( h_{y} \), length, \( L \), and strut thickness, \( t \), were fabricated to explore the failure modes identified in Section 2. Table 3 shows the dimen-

![Fig. 2. Failure modes in sandwich columns subjected to edge compression. (a) Euler macro-buckling; (b) core shear macro-buckling; (c) face sheet wrinkling; (d) face sheet microbuckling; (e) face sheet delamination.](image)

![Fig. 3. Schematic of the manufacturing molds used for fabrication of the carbon fiber pyramidal truss cores.](image)

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Failure criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber tensile failure ( (\sigma_{x} &gt; 0) )</td>
<td>( (\sigma_{x}XX)^{2} + (\sigma_{y}YY)^{2} + (\sigma_{z}ZZ)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Fiber compressive failure ( (\sigma_{x} &lt; 0) )</td>
<td>( (\sigma_{x}XX)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Matrix tensile cracking ( (\sigma_{yy} &gt; 0) )</td>
<td>( (\sigma_{yy}YY)^{2} + (\sigma_{zz}ZZ)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Matrix compressive cracking ( (\sigma_{yy} &lt; 0) )</td>
<td>( (\sigma_{yy}YY)^{2} + (\sigma_{zz}ZZ)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Fiber–matrix shear cracking ( (\sigma_{xy} &gt; 0) )</td>
<td>( (\sigma_{xy}XY)^{2} + (\sigma_{xz}XZ)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Delamination in tension ( (\sigma_{x} &gt; 0) )</td>
<td>( (\sigma_{x}XX)^{2} + (\sigma_{yz}YZ)^{2} \geq 1 )</td>
</tr>
<tr>
<td>Delamination in compression ( (\sigma_{x} &lt; 0) )</td>
<td>( (\sigma_{x}XX)^{2} + (\sigma_{yz}YZ)^{2} \geq 1 )</td>
</tr>
</tbody>
</table>

![Table 2. Stress-based three-dimensional Hashin criteria.](image)

- \( X_{P} \) and \( X_{C} \) are the tensile and compressive strength of fibers, \( Y_{T} \) and \( Y_{C} \) are the tensile and compressive strength of matrix, \( Z_{T} \) and \( Z_{C} \) are the tensile and compressive strength of the unidirectional laminate in the normal direction, \( \alpha_{t} \) and \( S_{t} \) is the component of the stress tensor and shear tensor defined in the classical form, where \( x \) and \( y \) are the in-plane axes.
Fig. 4. (a) Fabricated carbon fiber composite pyramidal truss structures with $p = 1.81\%$. (b) Sandwich panel with carbon fiber composite pyramidal truss core with
$p = 1.25\%$.

Table 3
Dimensions of the specimens and the predicted and measured failure loads and collapse modes. The failure modes in this table are abbreviated: FW (face wrinkling); FC (face
sheet crushing); CS (core shear macro-buckling).

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$r$ (kg/m$^3$)</th>
<th>$w$ (mm)</th>
<th>$L$ (mm)</th>
<th>$h_f$ (mm)</th>
<th>Analytical</th>
<th>Experiment</th>
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<tr>
<td></td>
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<td></td>
<td>Failure mode</td>
<td>Failure force (kN)</td>
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<tr>
<td>1</td>
<td>26.20</td>
<td>100</td>
<td>200</td>
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<td></td>
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<td>99</td>
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<td></td>
<td></td>
<td>FC</td>
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<tr>
<td>3</td>
<td>741.92</td>
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<td></td>
<td></td>
<td>FW</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>FC</td>
<td>107.21</td>
</tr>
</tbody>
</table>

Fig. 5. (a) The measured collapse response of sandwich column specimen 1. The specimen collapses in a core shear macro-buckling mode. The analytical prediction of the
collapse load is included. (b) Photographs of the deformation history of the panel in (a). (c) Debonding image of a typical node-face sheet cross-section A–A after macro elastic
buckling mode. (d) Strut fracturing image of a typical node-face sheet cross-section B–B after core shear macro-buckling mode.
sions of each set of sandwich panels, as well as the analytical predictions for each failure mode and the measured peak load and the observed failure mode. The details of the compression test and the results are discussed in the following section.

4. Compressive response of sandwich panels

The response of pyramidal core sandwich columns was measured using INSTRON 5500 following ASTM C365/C 364M-05.

![Graph of load vs. displacement](image)

**Fig. 6.** (a) The measured collapse response of sandwich column specimen 2. The specimen collapses in a face wrinkling mode. The analytical prediction of the collapse load is included. (b) Photographs of the deformation history of the panel in (a). (c) Debonding image of a typical node-face sheet cross-section A–A after face wrinkling.
The axial compression tests were carried out in the quasi-static regime with a nominal displacement rate of 0.5 mm/min. At least two tests were carried out for each column geometry to ensure the repeatability of the results.

4.1. Euler or core shear macro-buckling

According to the analytical study presented in Section 2.1, core shear buckling is the dominant elastic macro-buckling mode for sandwich panels with thick face sheets and thin struts. No practical laboratory scale column could be designed to probe the Euler buckling mode. The measured compressive response of specimen 1 (with face sheets $[0/+45/-45/90]^4$ and $L = 200$ mm, $h_f = 2.0$ mm, $h = 11.2$ mm, $t = 0.52$ mm, $\rho = 26.20$ kg/m$^3$) is depicted in Fig. 5, along with a montage of photographs showing the specimen deformation and fracture at different stages of loading (I: initial, II: prior to buckling, III: after buckling).

After the initial linear response, specimen 1 buckles and bends outward and the resisting force of the column decreases sharply. According to Eq. (3), the buckling load of the sandwich panel is $P_b \approx 43.08$ kN (note that for this sandwich panel $P_b \gg P_f$). The measured peak value is $\sim 35.41$ kN (standard deviation $\sim 3.17$ kN), about 18% lower than the theoretical prediction. The load–displacement curve drops sharply after core shear macro-buckling. The response of the panel at this stage of loading is governed by a combination of core shear buckling and debonding of the struts from the face sheets as can be seen in Fig. 5c. The debonding occurs since the bonding between the core and face sheet is not strong enough to transmit the constraints of the cores. In addition to debonding, fracture of the trusses was also observed prior to the overall failure of the panel, as shown in Fig. 5d.

4.2. Face wrinkling

Fig. 6a shows the response of specimen 2 with face sheets $[+45/-45/0/-45/+45]$ and $L = 99$ mm, $h = 12.8$ mm, $h_f = 0.58$ mm, $t = 0.8$ mm, and $\rho = 32.03$ kg/m$^3$. Fig. 6b shows the deformed configurations of the sandwich panel at different stages of loading.

![Graph showing the load-displacement curve for specimen 1.](image)

![Series of photographs showing the deformation and fracture stages of specimen 1.](image)
4.3. Face sheet crushing

Fig. 7a shows the measured compressive response of specimen 3 with face sheets [0°/+45°−/−45/90]_5 and L = 142 mm, h = 2.38 mm, t = 2.4 mm, h = 13.6 mm and ρ = 71.92 kg/m^3. Fig. 7b shows the deformed configuration of the sandwich panel at different stages of deformation. In this experiment, a progressive end-crushing of carbon fiber composite sandwich panels was observed. As the compressive load reaches the critical load associated with the face crushing, both face sheets get crushed at the bottom of the test fixture, Fig. 7c. Using the numerical method presented in Section 2.3, we estimated σ_{cr} = 317.22 MPa, for this panel configuration. The analytical estimate of the critical load based on Eq. (5) is ~107.21 kN. The measured peak value in Fig. 7a is 96.4 kN (standard deviation ~8.70 kN) and is about 10% lower than the analytical prediction. This discrepancy is attributed to the imperfections in the manufactured specimens, as well as the assumptions made in developing the simple analytical model presented in Section 2.3 and in estimating the microbuckling strength of the composite face sheet. The local delamination of the face sheet near the clamped ends of the panel reduces the overall strength of the sandwich panel. However, the sandwich panel regained its strength partially as the deformation was increased. After a second peak load of ~78 kN, the sandwich panel lost its integrity, as the core completely debonded from one of the face sheets, and each part of the panel bent outwards (Stage IV). Fig. 7d shows an image of the face sheet after core debonding, taken after the complete failure of the panel.

5. Concluding remarks

We studied the compressive response and failure of composite sandwich panels with pyramidal truss cores under axial compression. Our investigation complements the previous studies on the response and performance of lightweight sandwich panels with complex core construction and provides insight into the failure mechanisms of composite sandwich panels. In the analytical part of the study, three different failure modes were considered and theoretical models were presented to estimate the failure load associated with each mode. In the experimental part of the work, we used the analytical estimates to fabricate sandwich columns that allowed us to probe the three different failure regimes. The measured peak loads were in reasonable agreement with the analytical predictions. In general, after the initial peak load, the bond strength was one of the key limiting factors in the performance of the panels. Debonding between the core and face sheets were observed in all experiments, leading to a reduction in the load carrying capacity of the panels in the post-failure regime.

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